IDENTIFICATION OF THE HEAT LOAD OF A PLAIN BEARING IN A NONSTATIONARY FRICTION PERIOD

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The problem of restoring the heat load of a plain bearing in a nonstationary friction period is solved by temperature measurements within the bushing.

One of the most important service characteristics of bearings is the friction moment. Meanwhile, direct measurement of this quantity is fraught with significant difficulties of technological nature.

The energy expended on friction in a plain bearing is determined by the work of the friction moment. Different energy-conversion processes (in nonthermal form) such as emission of phonons (acoustic waves), photons (triboluminescence), electrons (exoelectron emission), etc., occur during the sliding of solids. However, these components are so small that an assumption about the transfer of all the energy into heat is ordinarily used [1].

The rate of heat liberation in the frictional contact zone can be represented as q = kpv.

Therefore, the friction moment correlates with the magnitude of the total heat liberation in the contact zone.

The possibility of recovering the heat liberation and the friction moment in the slider contact, respectively, by means of values of the bearing temperature which can be measured more simply as compared with the friction moment is examined in this paper.

Let us consider the bearing represented in Fig. 1. Sliding occurs over the contact surface between the elements 1 and 2; the bushing is connected rigidly to the bearing housing. The shaft 1 and the housing 3 are made of metal, while the bushing is made of an antifriction polymer or composite material.

An algorithm for solving the nonstationary problem of heat conduction for a plain bearing was proposed in [2]. Taking the same assumptions as in [2], we set up the problem of determining the heat load.

Find the total heat-liberation rate Q(t) and the corresponding temperature distribution T(r, φ, t) from the system

$$\frac{\partial T}{\partial r} = A(r)\frac{\partial^2 T}{\partial r^2} + B(r)\frac{\partial T}{\partial r} + C(r)\frac{\partial^2 T}{\partial \varphi^2}, \tag{1}$$

$$r_2 < r < r_4, \quad 0 < \varphi < \pi, \quad 0 < t \leqslant t_m;$$

when $|\varphi| \leqslant \varphi_0$

$$s_s \rho(r_1) c(r_1) \frac{\partial T_s(t)}{\partial t} + 2\pi r_1 \alpha_s (T_s(t) - T_0) = Q(t) + r_2 \lambda_1 \int_0^{\varphi_0} \frac{\partial T(r_2, \varphi, t)}{\partial r} d\varphi, \tag{2}$$

$$T(r_2, \varphi, t) = T_s(t); \tag{3}$$

when $|\varphi| > \varphi_0$

$$\lambda_1 \frac{\partial T(r_2, \varphi, t)}{\partial r} = \alpha(T(r_2, \varphi, t) - T_0), \tag{4}$$

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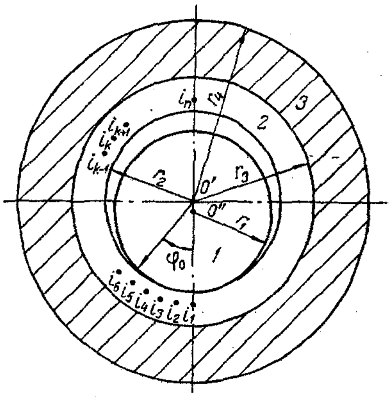

Fig. 1. Diagram of a friction unit: 1) shaft; 2) bushing; 3) housing. φ_0 is the contact angle between the shaft and the bushing, and i_k are temperature measurement points.

$$\lambda_2 \frac{\partial T(r_4, \varphi, t)}{\partial r} = -\alpha (T(r_4, \varphi, t) - T_0), \quad 0 \leqslant \varphi \leqslant \pi, \tag{5}$$

$$\frac{\partial T(r, 0, t)}{\partial \varphi} = \frac{\partial T(r, \pi, t)}{\partial \varphi}, \quad r_2 \leqslant r \leqslant r_4, \tag{6}$$

$$T(r, \varphi, 0) = T_0; \tag{7}$$

under known additional information for fixed R

$$T(R, \varphi, t) = f(\varphi, t); \quad r_2 < R < r_3. \tag{8}$$

The usual conjugate conditions are satisfied on the bushing–housing boundary

$$\lambda_1 \frac{\partial T(r_3 - 0, \varphi, t)}{\partial r} = \lambda_2 \frac{\partial T(r_3 + 0, \varphi, t)}{\partial r}, \quad T(r_3 - 0, \varphi, t) = T(r_3 + 0, \varphi, t);$$

and $A(r)$ $B(r)$, $C(r)$, $c(r)$, $\rho(r)$, α_s, α, T_0, and $f(\varphi, t)$ are known.

We replace the problem by an extremal problem on the minimum of the root-mean-square functional

$$J[Q(t)] = \int_0^{t_m} \int_0^{\pi} [T(R, \varphi, t) - f(\varphi, t)]^2 \, d\varphi dt \tag{9}$$

in solutions of the system (1)-(7).

It is natural to use one of the gradient methods that are applied extensively in solving similar problems [3-5].

To determine the gradient of the functional (9), we consider the problem conjugate to (1)-(7) [3]. The condition $|\varphi| \leqslant \varphi_0$ for $r = r_2 = r_s$ introduces definite difficulties in the derivation of the boundary condition for the conjugate problem. Thus, in contrast to [4], where a system of linear algebraic equations is obtained to match the values of the Lagrange multipliers on the boundary, we arrive at a system of integrodifferential equations from the necessary conditions for an extremum in our case

$$-s_s \rho(r_1) c(r_1) \frac{\partial \eta(r_2, \varphi, t)}{\partial t} + 2\pi r_1 \alpha_s \eta(r_2, \varphi, t) + r_2 \left[\frac{\partial}{\partial r} (A(r_2) \ \psi(r_2, \varphi, t)) - B(r_2) \psi(r_2, \varphi, t)\right] = 0, \tag{10}$$

$$\lambda_1 \int_0^{\varphi_0} \eta(r_2, \varphi, t) \, d\varphi + A(r_2) \psi(r_2, \varphi, t) = 0, \tag{11}$$

$\eta(r_2, \varphi, t)$ and $\psi(r_2, \varphi, t)$ are Lagrange multipliers.

Assuming $\eta(r_2, \varphi, 0) = 0$, we find $\eta(r_2, \varphi, t)$ from (10):

$$\eta(r_2, \varphi, t) = \frac{r_2 \exp(Pt)}{s_s \rho(r_1) c(r_1)} \int_0^t \left[\frac{\partial}{\partial r} (A(r_2) \psi(r_2, \varphi, \tau)) \exp(-P\tau) - B(r_2) \psi(r_2, \varphi, \tau)\right] d\tau, \tag{12}$$

where $P = 2\pi r_1 \alpha_s / (s_s \rho(r_1) c(r_1))$.

Substituting (12) into (11), we have

$$r_2\lambda_1 \exp(Pt) \int_0^{\varphi_0}\int_0^t \left[B(r_2)\psi(r_2, \varphi, \tau) - \frac{\partial}{\partial r}(A(r_2)\psi(r_2, \varphi, \tau))\right] \exp(-P\tau)\,d\tau = A(r_2)\psi(r_2, \varphi, t)\,s_s\rho(r_1)\,c(r_1). \quad (13)$$

Differentiating this latter with respect to t, we finally obtain

$$-A(r_2)\,s_s\,\rho(r_1)\,c(r_1)\frac{\partial\psi_s(r_2, \varphi, t)}{\partial t} + 2\pi r_1\alpha_s A(r_2)\,\psi_s(r_2, \varphi, t) + r_2\lambda_1\int_0^{\varphi_0}\left[B(r_2)\psi(r_2, \varphi, t) - \frac{\partial}{\partial r}(A(r_2)\psi(r_2, \varphi, t))\right] \times$$
$$\times\, d\varphi = 0. \quad (14)$$

Taking account of condition (13) the conjugate boundary-value problem is written as

$$-\frac{\partial\psi}{\partial t} = \frac{\partial^2}{\partial r^2}(A(r)\psi) - \frac{\partial}{\partial r}(B(r)\psi) + C(r)\frac{\partial^2\psi}{\partial\varphi^2} + 2\,[T(R, \varphi, t) - f]\,\delta(r - R), \quad (15)$$

$$r_2 < r < r_4, \quad 0 < \varphi < \pi, \quad 0 < t < t_m;$$

when $|\varphi| > \varphi_0$

$$B(r_2)\psi(r_2, \varphi, t) - \frac{\partial}{\partial r}(A(r_2)\psi(r_2, \varphi, t)) = -\frac{A(r_2)\,\alpha}{\lambda_1}\psi(r_2, \varphi, t), \quad (16)$$

$$B(r_4)\psi(r_4, \varphi, t) - \frac{\partial}{\partial r}(A(r_4)\psi(r_4, \varphi, t)) = \frac{A(r_4)\,\alpha}{\lambda_2}\psi(r_4, \varphi, t), \quad (17)$$

$$0 \leqslant \varphi \leqslant \pi,$$

$$\frac{\partial\psi(r, 0, t)}{\partial\varphi} = \frac{\partial\psi(r, \pi, t)}{\partial\varphi} = 0, \quad (18)$$

$$\psi(r, \varphi, t_m) = 0, \quad \delta(r - R) = \begin{cases}1, & \text{if} \quad r = R,\\ 0, & \text{if} \quad r \neq R.\end{cases} \quad (19)$$

Taking into account $B(r) = A(r)/r$ and $C(r) = A(r)/r^2$ and making the change of variable $\bar{\psi} = A\psi/r$, we obtain the system

$$-\frac{\partial\bar{\psi}}{\partial t} = A(r)\frac{\partial^2\bar{\psi}}{\partial r^2} + B(r)\frac{\partial\bar{\psi}}{\partial r} + C(r)\frac{\partial^2\bar{\psi}}{\partial\varphi^2} + \frac{2A(r)}{r}[T(R, \varphi, t) - f]\,\delta(r - R), \quad (20)$$

$$r_2 < r < r_4, \quad 0 < \varphi < \pi, \quad 0 < t < t_m;$$

when $|\varphi| \leqslant \varphi_0$

$$-s_s\,\rho(r_1)\,c(r_1)\frac{\partial\bar{\psi}_s(t)}{\partial t} + 2\pi r_1\alpha_s\bar{\psi}_s(t) = r_2\lambda_1\int_0^{\varphi_0}\frac{\partial\bar{\psi}(r_2, \varphi, t)}{\partial r}\,d\varphi; \quad (21)$$

when $|\varphi| > \varphi_0$

$$\lambda_1\frac{\partial\bar{\psi}(r_2, \varphi, t)}{\partial r} = \alpha\bar{\psi}(r_2, \varphi, t); \quad (22)$$

when $|\varphi| \leqslant \pi$

$$\lambda_2\frac{\partial\bar{\psi}(r_4, \varphi, t)}{\partial r} = -\alpha\bar{\psi}(r_4, \varphi, t); \quad (23)$$

$$\frac{\partial\bar{\psi}(r, 0, t)}{\partial\varphi} = \frac{\partial\bar{\psi}(r, \pi, t)}{\partial\varphi} = 0; \quad (24)$$

$$\bar{\psi}(r, \varphi, t_m) = 0. \quad (25)$$

Upon introducing the variable $\tau = t_m - t$ the conjugate problem (20)-(25) is solved by using the same calculational procedure as for problem (1)-(7).

The gradient of the functional (9) is found from the formula

$$J'[Q(t)] = \frac{r\cdot\bar{\psi}(r_2, \varphi, t)}{\lambda_1}. \quad (26)$$

The method of conjugate gradients, which has the best characteristics for solving incorrectly posed inverse problems [3, 4] as compared with the method of steepest descent, was selected as the numerical method for minimizing the functional (9).

Different functions of the time Q(t) under the following geometric dimensions: $r_1 = 12.5$ mm, $r_2 = 13$ mm, $r_3 = 14.5$ mm, $r_4 = 29.5$ mm were selected as the exact solution for the model problem. Steel was the material for the shaft and the race, and the filled fluoroplastic

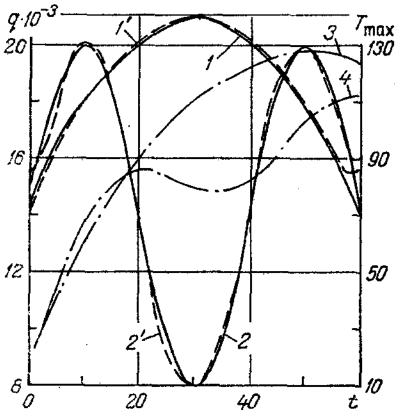

Fig. 2

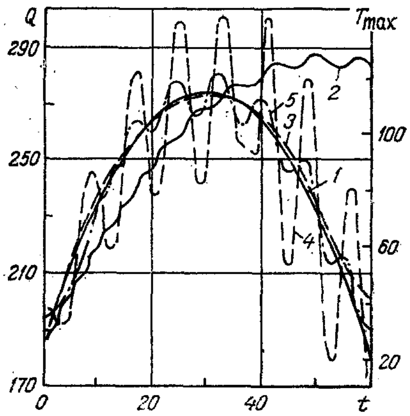

Fig. 3

Fig. 2. Results of a numerical experiment on recovering the specific heat-liberation rate q(t): 1, 2) given dependences of q(t): 1) $q(t) = 14 \cdot 10^3 + 8t^2 + 480t$; 2) $q(t) = 10^3(14 + 6\sin(\pi t/20))$; 3, 4) dependences of the maximal temperature $T_{max}(r, 0, t)$ for the functions 1 and 2 for q(t), respectively; 1', 2') the recovered functions q(t); t, min; q, W/m².

Fig. 3. Recovery of the total heat-liberation rate Q(t) by means of values of the temperature measured with an error: 1) the given function Q(t); 2) the temperature T_{max} measured with error; 3) Q(t) recovered by using a self-regulating algorithm; 4) solution of the inverse heat-conduction problem for large iteration numbers; 5) solution of the inverse heat-conduction problem obtained by the method of iterational regularization; Q, W/m².

F4K20 for the bushing, with $c\rho = 2.67 \cdot 10^6$ J/m³·°C and $\lambda = 0.39$ W/m·°C for the F4K20, and $c\rho = 3.48 \cdot 10^6$ J/m³·°C and $\lambda = 25.35$ W/m·°C for the steel, and α_s is determined for the shaft velocity v = 1 m/sec.

The total heat-liberation rate Q(t) is determined from the following formula:

$$Q(t) = 2r_1 \int_0^{\varphi_0} q(\varphi, t)\, d\varphi,$$

where $q(\varphi, t)$ is the specific heat-liberation rate. Computations were performed for

$$q(\varphi, t) = \begin{cases} q(t), & |\varphi| \leqslant \varphi_0 \\ 0 & |\varphi| > \varphi_0 \end{cases}$$

and, correspondingly, $Q(t) = 2r_2\varphi_0 q(t)$.

The temperature was given on the circle R = 13.6 m at the mesh nodes by the results of solving the direct problem. The solution of the inverse boundary-value problem is represented in Fig. 2 for the experimental information given exactly. The approximate solution is sufficiently close to the exact.

For perturbed initial data

$$T^*(\varphi, t) = T(\varphi, t) + \delta_0 w, \tag{27}$$

where $\delta_0 = 0.03T_{max}$ and w is a random variable with uniform distribution law ($-1 \leqslant w \leqslant 1$), the approximate solution is strongly oscillatory in nature for large iteration numbers (Fig. 3). This is the natural behavior of the iteration solution of an incorrectly posed inverse problem.

In this connection, it is expedient to halt the iteration process according to the condition [3]:

$$J[Q(t)] \leqslant \delta_T^2, \tag{28}$$

$$\delta_T^2 = \int_0^{t_m}\int_0^{\pi} \sigma^2(\varphi,\ t)\,d\varphi dt, \tag{29}$$

where $\sigma^2(\ ,\ t)$ is the variance of the function $T^*(\varphi,\ t)$. This condition is satisfied by the seventh iteration. The corresponding results are represented by curve 5 in Fig. 3.

In order to obtain a more exact smooth solution of the inverse problem, we use a self-regularizing algorithm [3]. The solution is sought in the class of continuously differentiable functions satisfying the condition

$$Q(t) = \int_0^t \frac{dQ}{d\tau}\,d\tau. \tag{30}$$

Introducing the function

$$q(t) = \int_t^{t_m} J'_Q(\tau)\,d\tau,$$

we obtain

$$J'_{Q'}(t) = q(t),\quad t\in[0,\ t_m]. \tag{31}$$

The iteration sequence is then constructed

$$Q'^{k+1}(t) = Q'^{k}(t) - \bar{\beta}_k\,\bar{\xi}^k(t), \tag{32}$$

where $\bar{\xi}^k(t) = J'_{\theta'}(t) + \bar{\gamma}_k\,\bar{\xi}^{k-1}(t)$ is the conjugate direction,

$$\bar{\gamma}_0 = 0;\quad \bar{\beta}_k : J[Q'^{k+1}] = \min_{\bar{\beta}\geqslant 0} J[Q'^k - \bar{\beta}J'^k_{Q'}].$$

Integrating (31), we finally obtain

$$Q^{k+1}(t) = Q^k(t) - \bar{\beta}_k \int_0^t \bar{\xi}^k(t)\,dt + C_1.$$

If the value of the desired function is not refined for $t = 0$, then $C_1 = 0$. If the value of the desired function at the left end is refined, then

$$C_1 = Q'^{k+1}(0) - Q'^k(0) = -\bar{\beta}_k\,\bar{\xi}^k(0) = \bar{\beta}_k(q^{k-1}(0) + \bar{\gamma}_k\,\bar{\xi}^{k-1}).$$

Application of this regularization procedure permits a sufficiently smooth solution to be obtained. The halt according to condition (28) is reached at the 35th iteration in this case. The corresponding results are represented by curve 3 in Fig. 3. For a three-dimensional mesh $N_r = 20$, $N_\varphi = 20$, $N_t = 60$ the machine time expended for execution of one iteration was 2 min for an M-4030 computer. Around 3 min is expended for a self-regularizing algorithm.

The results of numerical modeling show that the accuracy of recovering the heat-liberation rate Q(t) by the self-regularizing algorithm is commensurate with the accuracy of the assignment of the initial data.

The method proposed can be used for temperature diagnostics of the friction characteristics of machine and mechanism friction units.

NOTATION

k, friction coefficient; P, specific pressure; v, slip velocity; q, specific heat-liberation rate; Q(t), total heat-liberation rate in the frictional contact zone; T_0, initial temperature; T, bearing temperature; T_s, shaft surface temperature; t, running time; r, φ, polar coordinates; t_m, testing time; s_s, shaft cross-sectional area; ρ, density; c, specific heat; λ_1, heat conduction of the bushing material; λ_2, heat conduction of the shaft and race material; α_s, heat-elimination coefficient from the shaft surface; α, heat-elimination coefficient from the free surfaces of the bushing and race; η, ψ, Lagrange multipliers; $\bar{\beta}_k$, iteration parameter of the regularization algorithm; and N_r, N_φ, N_t, quantity of mesh nodes along the radius, angle, and time, respectively.

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STEADY-STATE HEAT CONDUCTION FOR A REGION BOUNDED BY A SPHERE AND A TANGENT PLANE

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It is shown that the problem of potential theory for a half-space with a spherical cavity with boundary conditions of the first and third kinds reduces to an ordinary differential equation which can be solved efficiently by numerical methods.

It is well known that boundary conditions of the third kind prevent the separation of variables in the general case for boundary-value problems of potential theory. However, as shown in [1, 2], bipolar coordinates in a plane can be used to solve certain problems involving off-center cylinders with a boundary condition of the third kind on the surface of one of the cylinders. In the case of contacting spheres, a system of degenerate bispherical coordinates can be used [3], in which the Fourier–Bessel integral transform method reduces the problem to an ordinary differential equation for the transform. We consider a similar case, when one of the spheres becomes a half-space.

Statement of the Problem. We consider the steady-state temperature distribution between a sphere and a tangent plane with the boundary conditions such that the sphere is at a given constant temperature and the plane is cooled according to Newton's law by a medium at zero temperature (Fig. 1).

In a system of degenerate bispherical corodinates (α, β, φ) related to cylindrical coorcinates (z, ρ, φ) by

$$z + i\rho = \frac{2Ri}{\alpha + i\beta}, \tag{1}$$

$$0 \leqslant \beta \leqslant 1, \quad 0 \leqslant \alpha \leqslant \infty, \quad -\pi \leqslant \varphi \leqslant \pi,$$

the equation of a sphere of radius R becomes $\beta = 1$, and the equation of the tangent plane will be $\beta = 0$. For the case of rotational symmetry, the problem reduces to the solution of Laplace's equation in the form

$$\frac{\partial}{\partial\alpha}\left(\frac{\alpha}{\alpha^2+\beta^2}\frac{\partial T}{\partial\alpha}\right) + \frac{\partial}{\partial\beta}\left(\frac{\alpha}{\alpha^2+\beta^2}\frac{\partial T}{\partial\beta}\right) = 0, \quad 0 < \beta < 1, \quad 0 \leqslant \alpha < \infty, \tag{2}$$

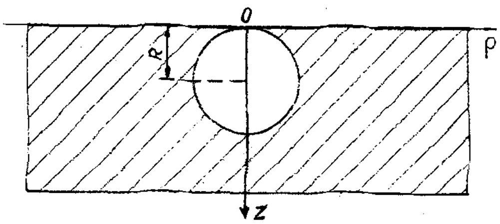

Fig. 1. Half-space with a spherical cavity.

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